Equilibrium Interest Rate and Tax

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We have previous solved the household's asset supply problem with a borrowing constraint. And also the firm's asset demand problem. We used first order taylor approximation to solve for the approximate equilibrium interest rate before for the firm's asset demand problem and for the households' savings problem without borrowing constraint. Here we find equilibrium interest rate with the constrained borrowing problem. I will analyze the effect of an interest rate (borrowing) rate subsidy for firms and borrowing households that is paid for by savings tax.

How do households with different β respond to changes in r given Borrowing Constraint?

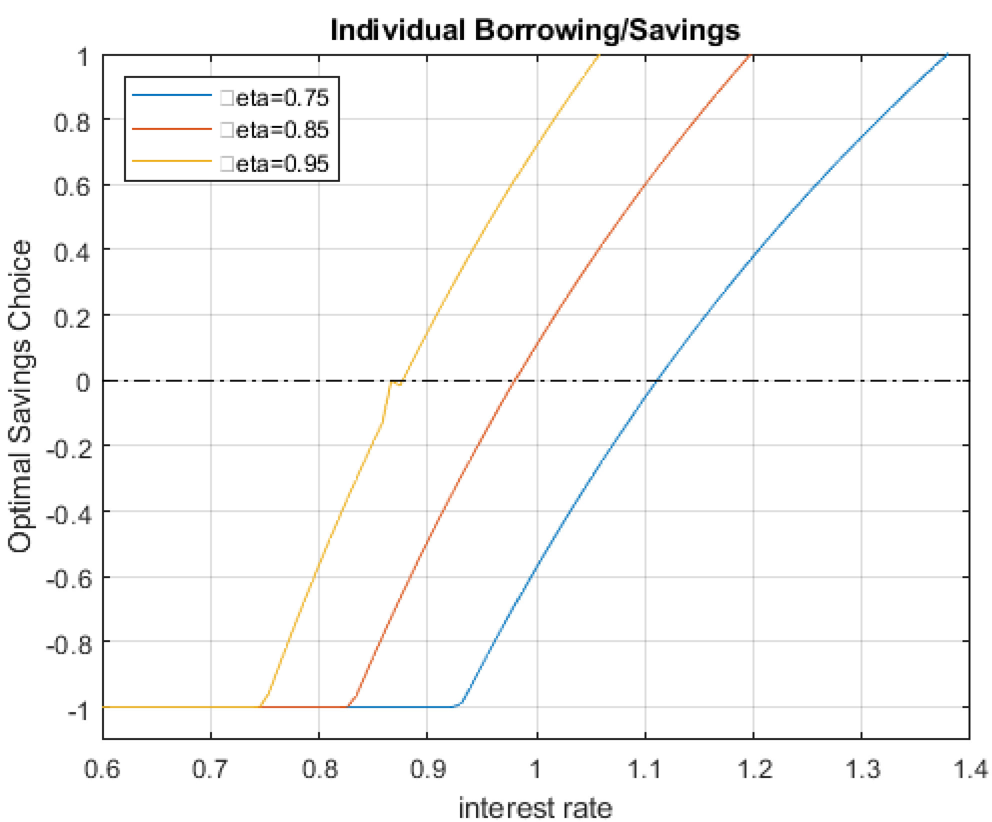
Following our previous discussions, the household's borrowing constrained problem is:

- specifically: $\max_b \log(Z_1 - b) + \beta_i \cdot \log(Z_2 + b \cdot (1 + r))$
- with: $b \geq \overline{b}$

I introduce now heterogeneity in β. There are $N = 3$ households, each with a different β_i. Note that lower β household at the same interest rate r will be more interested in borrowing rather than saving. The households have the same Z and face the same $\overline{b}$. Look at the graph below, at some interest rate, all three households want to borrow, at other rates, some want to borrow and others want to save.

```
clear all
% Parameters
z1 = 12;
z2 = 10;
b_bar_num = -1; % borrow up to 1 dollar

% Vector of 3 betas
beta_vec = [0.75 0.85 0.95];
% Vector of interest rates
r_vec = linspace(0.6, 1.40, 100);

% What we had from before to use fmincon
A = [-1];
q = -b_bar_num;
b0 = [0]; % starting value to search for optimal choice

% A vector to store optimal choices
rows = length(r_vec);
cols = length(beta_vec);
b_opti_mat = zeros(rows, cols);
% Solving for optimal choices as we change Z2
for j=1:1:length(beta_vec)
```

```
    for i=1:1:length(r_vec)
        U_neg = @(x) -1*(log(z1 - x(1)) + beta_vec(j)*log(z2 + x(1)*r_vec(i)));
        options = optimoptions('FMINCON','Display','off');
        [x_opti,U_at_x_opti] = fmincon(U_neg, b0, A, q, [], [], [], [], [], options);
        b_opti_mat(i, j) = x_opti(1);
    end
end
% Plot Results
legendCell = cellstr(num2str(beta_vec', '\beta=%3.2f'));
figure()
% Individual Demands at different Interest Rate Points
plot(r_vec, b_opti_mat)
ylim([-1.1 1]);
xlim([min(r_vec) max(r_vec)]);
hold on
plot(r_vec,ones(size(r_vec)) * 0, 'k-.');
grid on;
title('Individual Borrowing/Savings')
ylabel('Optimal Savings Choice')
xlabel('interest rate')
legend(legendCell, 'Location','northwest');
```



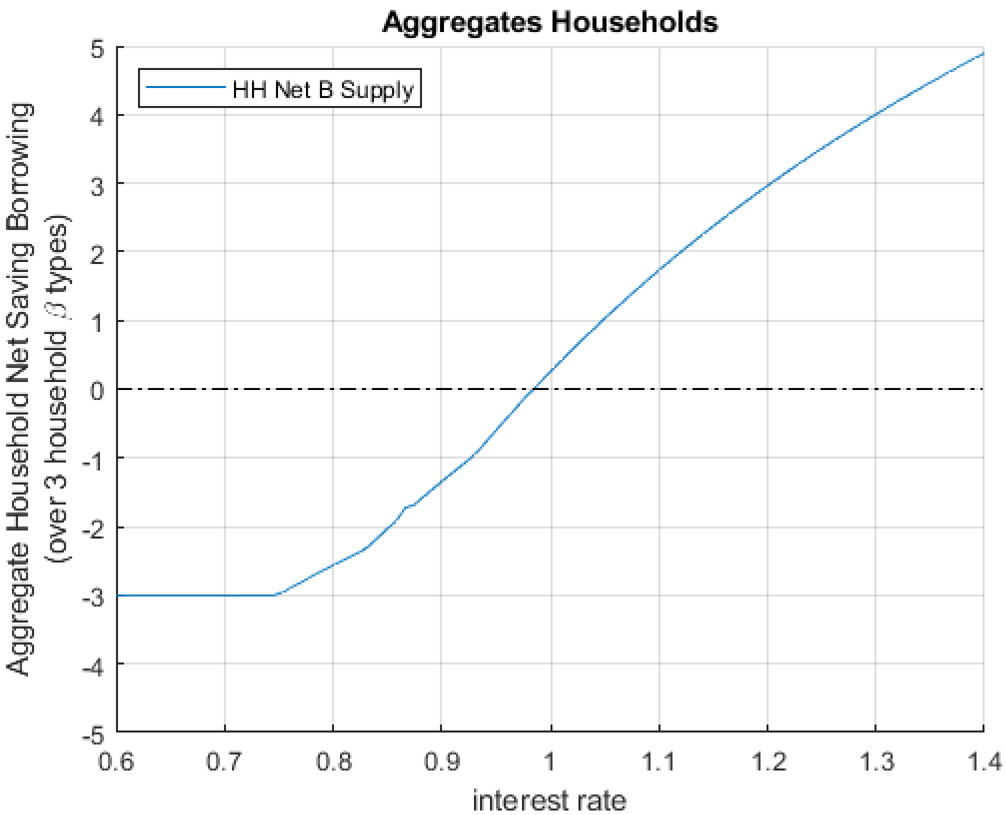
Aggregate Household Excess Supply along Interest Rate

When we solved for the equilibrium interest rate before, we had a firm that demanded credit and a household that supplied credit. Now we are more flexible, as shown in the chart above, households could be supplying or demanding credit. The equilibrium now is about clearing the aggregate demand and supply for the credit market considering both firms and households where households now could either be on demand or supply side. At a particular r, if households all want to borrow, there will be no lending, so that particular interest rate will not clear market. We will increase interest rate until some households are willing to save. Eventually, we find the market clearing interest rate.

If the economy has on the household side exactly these three households, we can sum the aggregate demand and supply for credit at each r from the households by summing across the $b^*(r,\beta_i)$. If households with these three different discount factors are in different proportions in the data for a particular country, we can sum up the weighted average.

- **Aggregate Household Excess Supply**: $B_{hh}^* = \sum_{i=1}^3 b^*(r,\beta_i)$

```
figure()
hold on;
% Aggregate demand (borrow meaning negative) and supply (saving positive) for households,
% just add sum (the 2 means sum over columns), it will sum across columns, each column is a dif
plot(r_vec, sum(b_opti_mat, 2))
plot(r_vec,ones(size(r_vec)) * 0, 'k-.');
ylim([-5 5]);
xlim([min(r_vec) max(r_vec)]);
grid on;
title('Aggregates Households')
ylabel({['Aggregate Household Net Saving Borrowing'], ['(over 3 household \beta types)']})
xlabel('interest rate')
legend({'HH Net B Supply'}, 'Location','northwest');
```



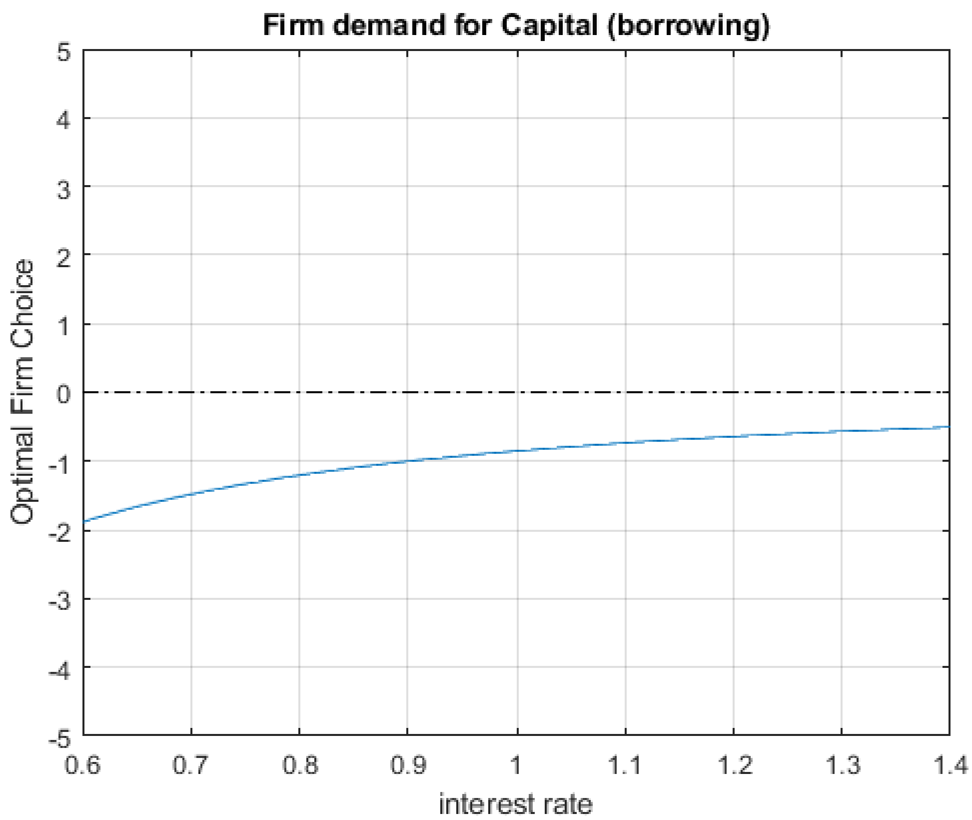
Firm Demand for Credit

We also have the aggregate Demand for the firm side based on the firm's capital only problem, with α_l for elasticity of labor, and α_k for elasticity of capital, and L is fixed at 1:

- **Firm Demand For Capital**: $K^*_{firm} = \left(\frac{r}{p \cdot A \cdot \alpha \cdot L^{\alpha_l}}\right)^{\frac{1}{\alpha_k - 1}}$

```
figure()
% Aggregate demand from firms (borrowing from firms)
p = 1;
A = 2.5;
alpha_K = 0.36;
alpha_L = 0.5;
L = 1;
FIRM_K = (r_vec./(p*A*alpha_K*(L^alpha_L))).^(1/(alpha_K-1));
% Individual Demands at different Interest Rate Points
plot(r_vec, (-1)*FIRM_K)
ylim([-5 5]);
xlim([min(r_vec) max(r_vec)]);
hold on
plot(r_vec,ones(size(r_vec)) * 0, 'k-.');
grid on;
title('Firm demand for Capital (borrowing)')
```

```
ylabel('Optimal Firm Choice')
xlabel('interest rate')
```



Economy Wide Excess Supply for Credit (Firm + Households)

The firm is demanding credit (it is borrowing), so we put a negative sign in front of K demanded:

- **Economy-wide excess supply of Credit**: $\text{ExcesCreditSupply}(r) = B^{*}_{hh}(r) - K^{*}_{firm}(r)$
- If the term above is positive that means total saving from households is greater than total borrowing from households and firms.

Equilibrium interest rate is the interest rate where excess credit supply is equal to zero:

- to find **equilibrium interest rate**: find r^{equi}, where at this interest rate: $B^{*}_{hh}(r^{equi}) - K^{*}_{firm}(r^{equi}) = 0$

```
% Summing up to get excess credit supply
excess_credit_supply = (sum(b_opti_mat, 2) + (-1)*FIRM_K');
% find at which interest rate we are closest to zero
[excess_credit_supply_at_equi, equi_idx_in_rvec] = min(abs(excess_credit_supply));
equilibrium_r = r_vec(equi_idx_in_rvec);
lowbeta_hh_b_equi = b_opti_mat(equi_idx_in_rvec, 1);
midbeta_hh_b_equi = b_opti_mat(equi_idx_in_rvec, 2);
highbeta_hh_b_equi = b_opti_mat(equi_idx_in_rvec, 3);
FIRM_K_equi = -FIRM_K(equi_idx_in_rvec);
results_withno_tax = table(equilibrium_r, excess_credit_supply_at_equi, equi_idx_in_rvec, FIRM_
```

```
disp(results_withno_tax)
```

equilibrium_r	excess_credit_supply_at_equi	equi_idx_in_rvec	FIRM_K_equi	lowbeta_hh_b_equi	midbeta
1.0364	0.022569	55	-0.80217	-0.37092	0.

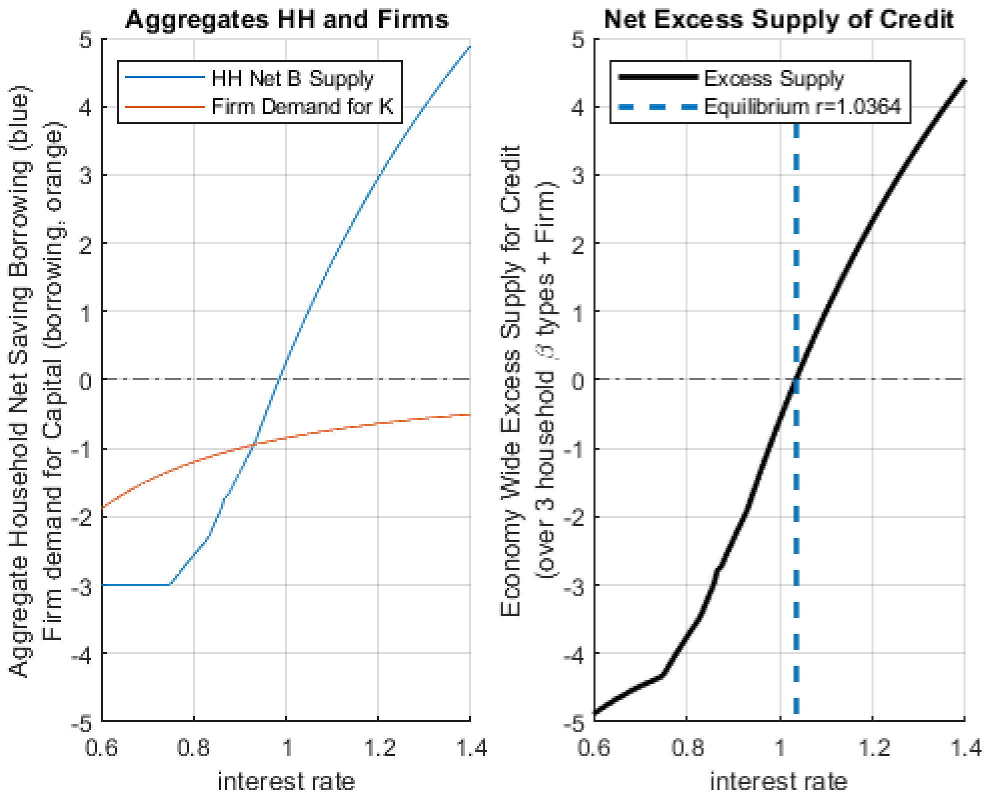
Note that our equilibrium is an approximation, because we only had a grid of r, the excess total supply is 0.023, which is close to 0, but not actualy 0. The numbers above show that our equilibrium interest rate is approximately 1.036, and at this equilibrium out of our three households and firm:

1. the firm borrows: 0.80217 (this is K, based on which we can find total output Y)
2. household 1 borrow: 0.37
3. household 2 saves: 0.30
4. household 3 saves: 0.89

These sum up to approximately 0. Note that none of our three households is borrowing constrained at the equilibrium. We can redraw the chart earlier and show the aggregate demand and supply for credit B_{hh} from the household side:

```
figure()
subplot(1,2,1)
hold on;
% Aggregate demand (borrow meaning negative) and supply (saving positive) for households,
% just add sum (the 2 means sum over columns), it will sum across columns, each column is a dif
plot(r_vec, sum(b_opti_mat, 2))
plot(r_vec, (-1)*FIRM_K)
plot(r_vec,ones(size(r_vec)) * 0, 'k-.');
ylim([-5 5]);
xlim([min(r_vec) max(r_vec)]);
grid on;
title('Aggregates HH and Firms')
ylabel({['Aggregate Household Net Saving Borrowing (blue)'], ['Firm demand for Capital (borrowi
xlabel('interest rate')
legend({'HH Net B Supply', 'Firm Demand for K'}, 'Location','northwest');
subplot(1,2,2)
hold on;
% Total Aggregate Net Demand for Credit at each R
plot(r_vec, excess_credit_supply, 'k', 'LineWidth', 2);
% Plot equilibrium interest rate line
plot(equilibrium_r*ones(1,10), linspace(min(excess_credit_supply), max(excess_credit_supply),10
% Zero line
plot(r_vec,ones(size(r_vec)) * 0, 'k-.');
ylim([-5 5]);
xlim([min(r_vec) max(r_vec)]);
grid on;
title('Net Excess Supply of Credit')
ylabel({['Economy Wide Excess Supply for Credit'], ['(over 3 household \beta types + Firm)']})
xlabel('interest rate')
```

```
legend({'Excess Supply', ['Equilibrium r=' num2str(equilibrium_r)]}, 'Location','northwest');
```



Demand and Supply for Credit with a Tax on Interest Rate

Suppose some government officials thinks we need to subsidize borrowing. They want to make it easier for households to borrow and also for firms to borrow. This sounds fantastic. Because if borrowing rate is lower for firm, the firm can borrow more in physical capital and increase output. How to pay for it? The officials decide to pay for it by taxing savings. Perhaps people with too much savings can take a cut on their interest rate earnings.

For the firm, this is just a discount on the borrowing rate. For households, this means if you save, you get your principle back next period, but you only get $1-\tau$ fraction of the interest rate earning. But if you borrow, you only pay $1-\tau$ fraction of your interest, rather than the full amount. Our problem remains the same as before, except that we need to resolve given the tax rate now. Let's apply the discount in borrowing to both households that borrow and firms that borrow:

Household problem with borrowing discount and saving tax:

- $\max_{b} \log(Z_1 - b) + \beta_i \cdot \log(Z_2 + b + b \cdot (r)(1-\tau))$

Firm optimal Policy function with borrowing discount:

- $$K^*_{firm} = \left(\frac{r \cdot (1-\tau)}{p \cdot A \cdot \alpha \cdot L^{\alpha_l}}\right)^{\frac{1}{\alpha_k - 1}}$$

Note: this policy **pays for itself** because the credit market clears, so government income from the savings tax will pay for exactly its subsidy on borrowing.

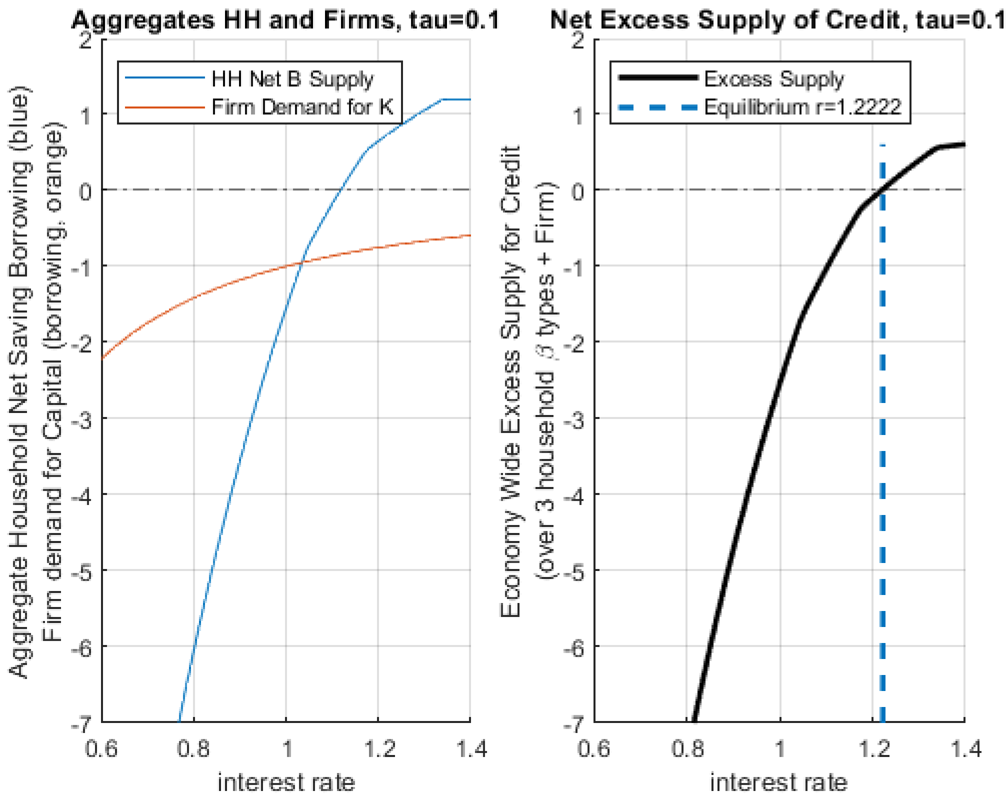
Suppose $\tau = 0.10$, let's solve for the new optimal choices and equilibrium given this tax policy. We re-use our previous codes but include now the tax:

```
tau = 0.10;
% Households' problem with Interest Tax and Subsidy
% A vector to store optimal choices
b_opti_mat = zeros(rows, cols);
% Solving for optimal choices as we change Z2
for j=1:1:length(beta_vec)
    for i=1:1:length(r_vec)
        U_neg = @(x) -1*(log(z1 - x(1)) + beta_vec(j)*log(z2 + x(1)*r_vec(i)*(1-tau)));
        options = optimoptions('FMINCON','Display','off');
        [x_opti,U_at_x_opti] = fmincon(U_neg, b0, A, q, [], [], [], [], [], options);
        b_opti_mat(i, j) = x_opti(1);
    end
end

% Firm's problem with interest tax and subsidy
FIRM_K = ((r_vec*(1-tau))./(p*A*alpha_K*(L^alpha_L))).^(1/(alpha_K-1));

% Approximate equilibrium
excess_credit_supply = (sum(b_opti_mat, 2) + (-1)*FIRM_K');
[excess_credit_supply_at_equi, equi_idx_in_rvec] = min(abs(excess_credit_supply));
equilibrium_r = r_vec(equi_idx_in_rvec);

% Grab Results
lowbeta_hh_b_equi = b_opti_mat(equi_idx_in_rvec, 1);
midbeta_hh_b_equi = b_opti_mat(equi_idx_in_rvec, 2);
highbeta_hh_b_equi = b_opti_mat(equi_idx_in_rvec, 3);
FIRM_K_equi = -FIRM_K(equi_idx_in_rvec);
results_with_tax = table(equilibrium_r, excess_credit_supply_at_equi, equi_idx_in_rvec, FIRM_K_
results_table = [results_withno_tax;results_with_tax];
results_table.Properties.RowNames = {'no r tax/subsidy', ['r tax/subsidy tau=' num2str(tau)]};
disp(results_table)
```

	equilibrium_r	excess_credit_supply_at_equi	equi_idx_in_rvec	FIRM_K_equi	low
no r tax/subsidy	1.0364	0.022569	55	-0.80217	
r tax/subsidy tau=0.1	1.2222	0.017077	78	-0.73085	

The table above compares the results with the tax and without. With the tax, the approximate equilibrium interest rate has to be higher because with the tax rate at the previous interest rate more people want to borrow (given the borrowing discount) and less people want to save (given the tax). To find the point where demand equals supply, interest rate increases to incentivize households to save despite the tax. In equilibrium, we now have a much higher interest rate that clears that market. Note that compared to before, firms are borrowing less, so output is now lower due to the higher equilibrium interest rate. The policy is potentially having the opposite of its intended effect. We solve for the general equilibrium effects of policies to help us think about these unintended consequences of policies.

```
figure()
subplot(1,2,1)
hold on;
% Aggregate demand (borrow meaning negative) and supply (saving positive) for households,
% % just add sum (the 2 means sum over columns), it will sum across columns, each column is a c
plot(r_vec, sum(b_opti_mat, 2))
plot(r_vec, (-1)*FIRM_K)
plot(r_vec,ones(size(r_vec)) * 0, 'k-.');
ylim([-7 2]);
xlim([min(r_vec) max(r_vec)]);
grid on;
title(['Aggregates HH and Firms, tau=' num2str(tau)])
ylabel({['Aggregate Household Net Saving Borrowing (blue)'], ['Firm demand for Capital (borrowi
xlabel('interest rate')
legend({'HH Net B Supply', 'Firm Demand for K'}, 'Location','northwest');
subplot(1,2,2)
hold on;
% Total Aggregate Net Demand for Credit at each R
plot(r_vec, excess_credit_supply, 'k', 'LineWidth', 2);
% Plot equilibrium interest rate line
plot(equilibrium_r*ones(1,10), linspace(min(excess_credit_supply), max(excess_credit_supply),10
% Zero line
plot(r_vec,ones(size(r_vec)) * 0, 'k-.');
ylim([-7 2]);
xlim([min(r_vec) max(r_vec)]);
grid on;
title(['Net Excess Supply of Credit, tau=' num2str(tau)])
ylabel({['Economy Wide Excess Supply for Credit'], ['(over 3 household \beta types + Firm)']})
xlabel('interest rate')
legend({'Excess Supply', ['Equilibrium r=' num2str(equilibrium_r)]}, 'Location','northwest');
```


Aggregates HH and Firms, tau=0.1
HH Net B Supply
Firm Demand for K
Aggregate Household Net Saving Borrowing (blue)
Firm demand for Capital (borrowing, orange)
interest rate
Net Excess Supply of Credit, tau=0.1
Excess Supply
Equilibrium r=1.2222
Economy Wide Excess Supply for Credit
(over 3 household β types + Firm)
interest rate